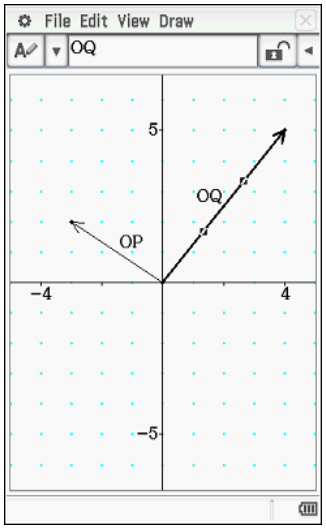
Activity 10 Position vectors and internal division

Aim: Develop the concept of position vectors and investigate their applications.

On the ClassPad Geometry application screen (right), position vectors $\overrightarrow{OP} = -3\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OQ} = 4\mathbf{i} + 5\mathbf{j}$ are shown.



1. Replicate the diagram on your ClassPad.

2. Determine the vector given by $\overrightarrow{OQ} - \overrightarrow{OP}$ (ie. $-\overrightarrow{OP} + \overrightarrow{OQ}$). This vector represents $\overrightarrow{PQ}$ (or the position vector of Q relative to P). Construct this vector on your ClassPad.

3. Determine the position vector of the point T that divides $\overrightarrow{PQ}$ in the ratio 1:3 (i.e. $\frac{1}{4}$ of the way along $\overrightarrow{PQ}$).

4. At 2 pm container ship *Andromeda* is at a point with position vector [–3, 7] km relative to a port. It is travelling with constant velocity of [12, 9] km/h. Determine:

   a) the position vector of the ship at 4:30 pm

   b) the distance of the ship from port at 4:30 pm

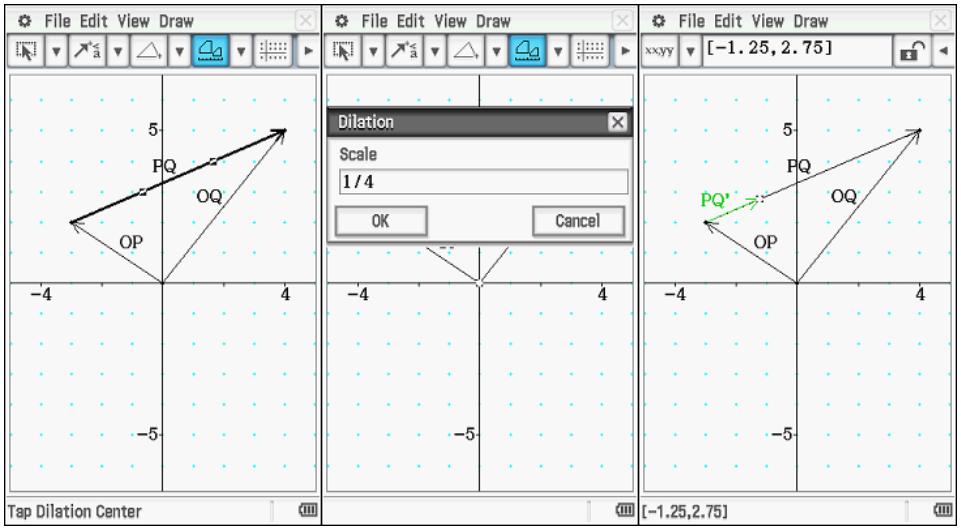
   c) the time, correct to the nearest 10 minutes, at which the ship is 75 km from the port. (CAS may be useful for this).

Learning notes

The location of a point in the plane can be described using co-ordinates, for example, point P(–3, 2), or as a position vector, for example, $\overrightarrow{OP} = -3\mathbf{i} + 2\mathbf{j}$ where O is the origin.

The labels on the vectors can be changed by selecting a vector then selecting label [A] from the Measure pull down menu.

Points that divide vectors in a given ratio can be found using the dilation [ ] tool in the Construct pull down menu. Select the required vector, then the dilation tool, then tap the tail of the vector as the dilation centre. The ratio written as a fraction of the total length is the scale factor.



Extension: vectors can also be "divided" externally simply by entering a ratio greater than one as the dilation factor.

A similar idea can be used for Q4. The initial position vector **r** and the constant velocity vector **s** are constructed. A dilation of factor 2.5 ($2\frac{1}{2}$ hours of travel) gives the position vector of the ship at 4:30 pm.

The distance can be found by selecting the tail of **r** and the head of **s**.